It should be observed that the series converges only for values of x such that $x^3 - 3x > 2$. But $x^3 - 3x + 2 = (x + 1)^2(x - 2)$. Hence x must be greater than 2.

Also solved by A. G. CARIS, F. M. MORGAN, HORACE OLSON, ELIJAH SWIFT, S. W. REAVES, and T. M. BLAKESLEE.

402. Proposed by R. D. CARMICHAEL, Indiana University.

Obtain other series similar to that of Borda given in the preceding problem (No. 401).

SOLUTION BY S. W. REAVES, University of Oklahoma.

If in the well-known series

$$(1)\quad \log(n+1) = \log n + 2\left[\frac{1}{2n+1} + \frac{1}{3}\left(\frac{1}{2n+1}\right)^3 + \frac{1}{5}\left(\frac{1}{2n+1}\right)^5 + \cdots\right]$$

we set

$$n = \frac{(x-2k)(x+k)^2}{4k^3},$$

and therefore

$$n + 1 = \frac{(x+2k)(x-k)^2}{4k^3}, \quad \text{and} \quad \frac{1}{2n+1} = \frac{2k^3}{x^3 - 3k^2x},$$

we shall have a generalized Borda's series

$$(2)\quad \begin{aligned} \log(x+2k) &= 2\log(x+k) - 2\log(x-k) + \log(x-2k) \\ &\quad + 2\left[\frac{2k^3}{x^3-3k^2x} + \frac{1}{3}\left(\frac{2k^3}{x^3-3k^2x}\right)^3 + \frac{1}{5}\left(\frac{2k^3}{x^3-3k^2x}\right)^5 + \cdots\right]. \end{aligned}$$

For $k = 1$, (2) becomes Borda's series.

Again, if in (1) we set

$$n = \frac{(x-3k)(x+k)^3}{16k^3x},$$

we shall have

$$(3)\quad \begin{aligned} \log(x+3k) &= 3\log(x+k) - 3\log(x-k) + \log(x-3k) \\ &\quad + 2\left[\frac{8k^3x}{x^4 - 6k^2x^2 - 3k^4} + \frac{1}{3}\left(\frac{8k^3x}{x^4 - 6k^2x^2 - 3k^4}\right)^3 + \cdots\right]. \end{aligned}$$

Also solved by F. M. MORGAN, T. M. BLAKESLEE, and A. M. HARDING.

GEOMETRY.

410. Proposed by A. H. HOLMES, Brunswick, Maine.

Given a focus and two tangents to an ellipse, prove that the locus of the foot of the normal corresponding to either tangent is a straight line.

SOLUTION BY A. M. HARDING, University of Arkansas.

Let SP and SP' be the fixed tangents, F the fixed focus, F' the other focus, and N the foot of the normal corresponding to SP. Draw FE and $F'E'$ perpendicular to SP, and FG and $F'G'$ perpendicular to SP'. Draw FF' cutting SP in L.